ON THE VISUALIZATION OF BLOWING UPS OF THE PLANE IN POINTS

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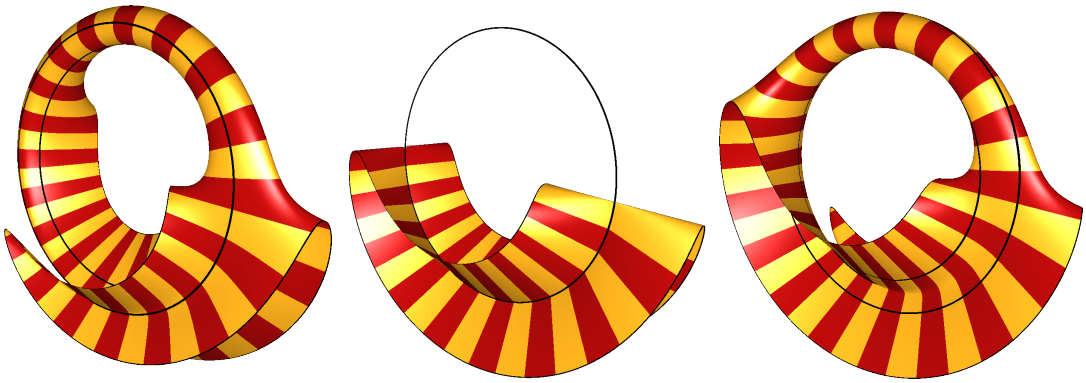
In the present talk we investigate the blowing up of a set of finite points $X = V(f, g) \subset \mathbb{A}^2_k, k$ a field. The technique of blowing ups is an important feature for the desingularization of algebraic varieties (see [Hironaka(1964)]). First we introduce the basic notions in the general case. Afterwards we illustrate the blowing ups in the plane.

The reason for that is the following: Mathematical models gain an increasing interest as a source of inspiration for applications in Mathematics, architecture etc. Most of the mathematical institutes of "traditional" german universities hold a collection of such mathematical models (see [Fischer(1986)]). With the aid of RealSurf (a graphics tool developed in the working group of the speaker (see [Stussak(2007)]) it is now possible to provide interactive visualizations of implicitely given surfaces in realtime.

In the talk it will be shown how to use these methods in order to visualize blowing ups of points X in the affine plane $\mathbb{A}^2_{\mathbb{R}}$ done by an embedding in a torus. Brodmann suggested a rational parametrization of this toroidal blowup (see [Brodmann(1995)]) . But its visualization fails in the neighborhood of X because the parametrization tends to indefinite terms of the form $\frac{0}{0}$. The approach presented in the talk is based on implicitization of the parametric form. By methods from commutative algebra there is a reduction of the implicitization to the computation of a resultant. By applying additional clipping techniques to the implicit surface we are able to visualize the toroidal blowup as well as its deformations by several parameters interactively in real-time. The methods provide



(A) $a < 0$ (B) $a = 0$ (C) $a > 0$

FIGURE 1. Visualization of the toroidal blowup of $\{(a, \pm a), (2a, 0)\}$ as the intersection of $f(u, v) = au + v^2 - 2a^2$, $g(u, v) = (u - 2a)(u - a)$. For $a = 0$ the double point and the two simple points collapse into a fourfold point.

insights in the structure of blowups of points, even if the points are interactively moved or tend to degenerations.

References

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